Physics I
ISI B.Math
Semestral Exam : December 3, 2010

Total Marks: 100. Time: 3 hours
Answer All Questions. Please note that you can use any result mentioned in the question paper to answer any other question

Question 1A. Total Marks: 2 X 6 = 12

How many degrees of freedom do the following systems have? (that is, how many independent coordinates are required to uniquely determine its position in space at a given instant). Justify your answer with one or two line calculations.

(a) A rigid body in pure rotation about an axis that is fixed in space
(b) A rigid body in translation and without any rotation
(c) A rigid body that can move along and rotate around a fixed axis
(d) A rigid body in two-dimensional planar motion. (Planar motion consists of translations parallel to a fixed plane and rotation about an axis perpendicular to that plane.)
(e) Two point particles connected by a rigid rod
(f) Two point particles connected by a compressible spring

Question 1B. Total Marks:8

Two identical rigid thin disks of arbitrary shape and total mass M are rotating with the same angular velocity Ω. One disk is rotating around a fixed axis that gores through the CM. The other one is rotating around a fixed axis parallel to the previous axis and a distance R away from the it. Which one has more kinetic energy, why and by how much? Assume that the space orientations of both disks are the same.

Question 2. Total Marks: 5+5+10=20

a. Show that in one dimensional elastic collision, the relative velocity after the collision is the negative of the relative velocity before the collision.

b. Using the above result (or otherwise) show that if a mass M with speed u collides elastically head on (one dimensional elastic collision) with a particle mass m with speed v, then in the limit $m/M \to 0$, the speed of the smaller mass after collision is v+2 u.

c. Using the above result show that if a basket ball is released from a height h above a horizontal floor, followed closely by a much smaller tennis ball (see Figure), then as the basketball bounces from the floor and collides with the tennis ball, the tennis ball will rise up to a height H approximately given by $H = 9h + d$ where d is the diameter of the basket ball. (see Figure 1). Assume all collisions to be elastic and one dimensional.

Question 3. Total Marks:4+4+4+4+4=20

A rigid hoop of mass M and radius R is rolling down an inclined plane with angle of inclination α.

a. Calculate the moment of inertia of the hoop and show that it is equal to MR^2.

b. Calculate the total kinetic energy at the instant when the center of mass has a translation velocity v (parallel to the slope).

c. List all the forces in the system and show that none of the forces except gravity does any work.

d. Write down the energy equation and show that the acceleration of the hoop is given by $\frac{1}{2}g \sin \alpha$.

e. If instead of the hoop we have a uniform disc of same mass M and radius R, which one will have greater acceleration and why?

Question 4. Total Marks:5+5+10=20

Write the Lagrangian and derive the equations of motion in the following systems using coordinates given in the figures.

a. The spring pendulum

b. Pendulum with a free support

c. A block of mass m sliding on a smooth edge of mass M which is sliding on a frictionless table.

In this case, find the expression for $\ddot{x}$ and $\ddot{y}$ in terms of m, M , g and the angle α. Show that in the limit M going to infinity, we get the familiar results for $\ddot{x}$ and $\ddot{y}$ of a block sliding down a rigid slope.

Question 5. Total Marks:4+6+6+4=20

The Lagrangian of a double pendulum of lengths l_1 and l_2 with masses M and m is given at the end of the question paper.

a. Show that the equations of motion for small oscillations around equilibrium are given by:

$$(M+m)l_1\ddot{\theta} + ml_2\ddot{\phi} + (M+m)g\theta = 0$$

and

$$l_1\ddot{\theta} + l_2\ddot{\phi} + g\phi = 0$$

Now assume $l_1 = l_2 = l$ and $M = 3\ m$ for the next three problems .

b. Find the frequencies of normal modes of oscillation in the case

c. Find the solution for θ and ϕ for the slow and the fast mode and sketch them.

d. What is the most general solution for θ, ϕ?

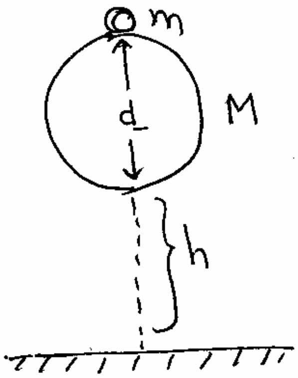
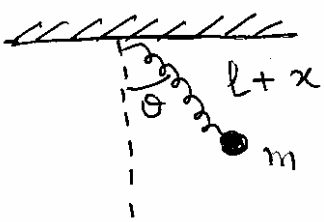


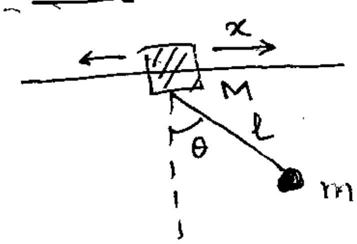
Fig. 1.



l: normal length of the spring.

x, θ: coordinates.

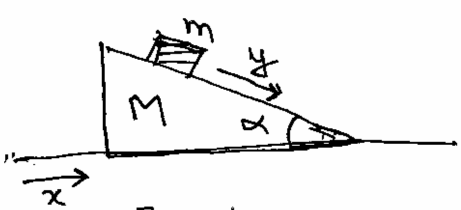
Fig. 2.



M can slide without friction.

x, θ: coordinates.

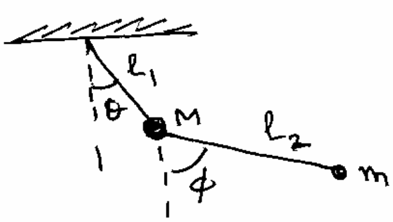
Fig. 3.



m, M slide without friction.

x, y: coordinates.

Fig. 4.



$$\mathcal{L} = T - V$$

$$T = \frac{1}{2} M (l_1 \dot{\theta})^2 + \frac{1}{2} m\left[(l_1 \dot{\theta}_1)^2 + (l_2 \dot{\phi})^2 + 2 l_1 l_2 \dot{\theta} \dot{\phi} \cos(\theta - \phi)\right]$$

$$V = (m+M) g l_1 (1 - \cos\theta) + m g l_2 (1 - \cos\phi)$$

Fig. 5.